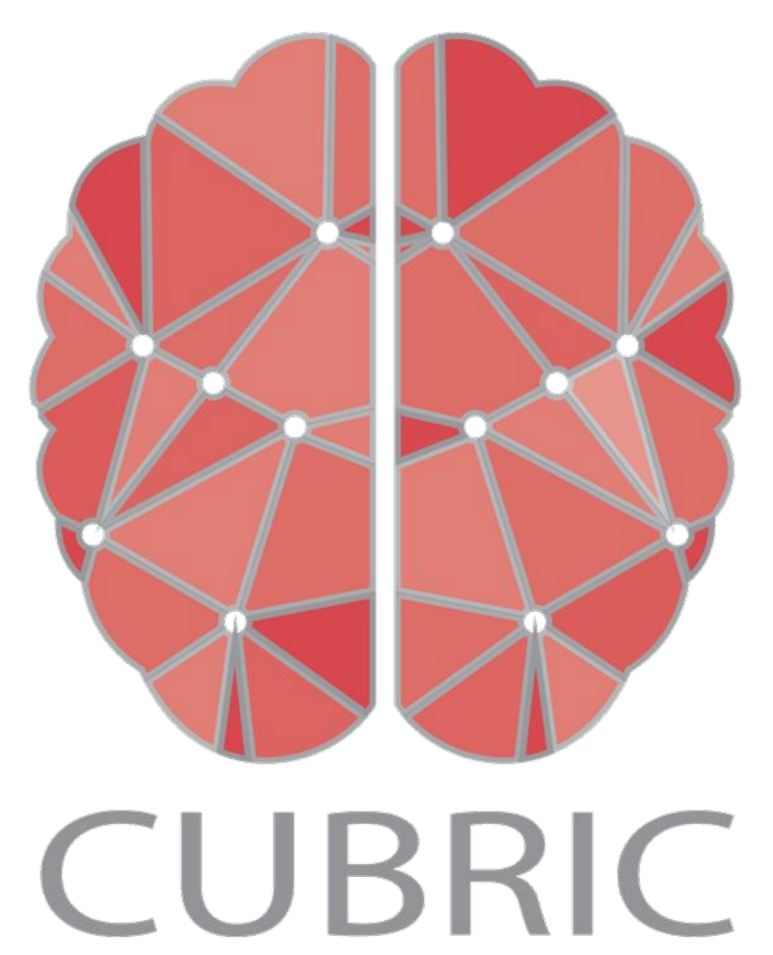


Reduced Order Modelling in EIT for the Efficient Estimation of the Electrical Conductivities of Head Tissue

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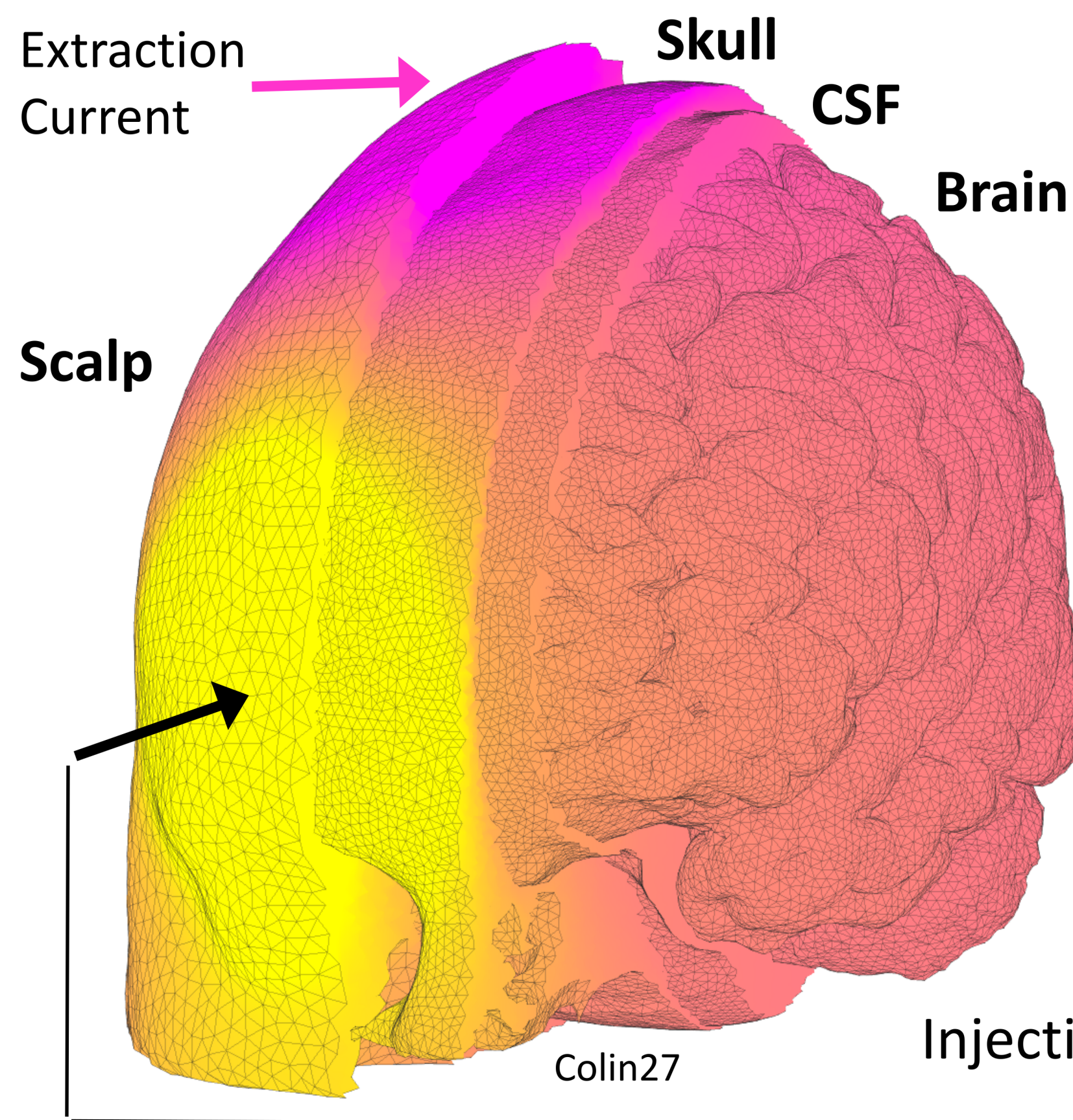
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Introduction: The Problem

It is well known in the field of EEG that accurate tissue conductivities used for an individual's head model make a significant improvement to the accuracy of source localization. It has also been shown that the conductivities of certain tissues vary substantially within the population [1]. EIT is a non-invasive imaging method used to estimate these conductivities however can be very computationally expensive. To reduce the computational load of parametrized PDE's (such as the forward problem in EIT), reduced order modelling (ROM) can be used. To date there has been no mathematically rigorous application of ROM to EIT. Our objective is to provide a framework for ROM applied to EIT and demonstrate its potential use in the field.



The governing equations:

Elements = 3 808 712
Nodes = 600 910

$$\begin{aligned} \nabla \cdot (\sigma \nabla u) &= 0 \quad \text{in } \Omega \\ \sigma \frac{\partial u}{\partial \mathbf{n}} &= \langle \sigma \nabla u, \mathbf{n} \rangle = 0 \quad \text{on } \partial\Omega \setminus \bigcup_{l=1}^L e_l \\ u + z_l \sigma \frac{\partial u}{\partial \mathbf{n}} &= U_l \quad \text{for } l = 1, \dots, L \\ \int_{e_l} \sigma \frac{\partial u}{\partial \mathbf{n}} d(\partial\Omega) &= I_l \quad \text{on } e_l \end{aligned}$$

Electric Potential

Weak formulation and Galerkin FEM

Conductivity Current Measurement at electrode

$$Au = b$$

The system of equations needed to be solved for the Forward Problem. $\mathbf{K}$ is called the Stiffness matrix where each dimension equals the number of elements in the FEM mesh.

$$\hat{\sigma} = \min_{\sigma} \{ (\hat{Y} - MK^{-1}(\sigma)b)^T (\hat{Y} - MK^{-1}(\sigma)b) \}$$

Maximum Likelihood Estimator [2]

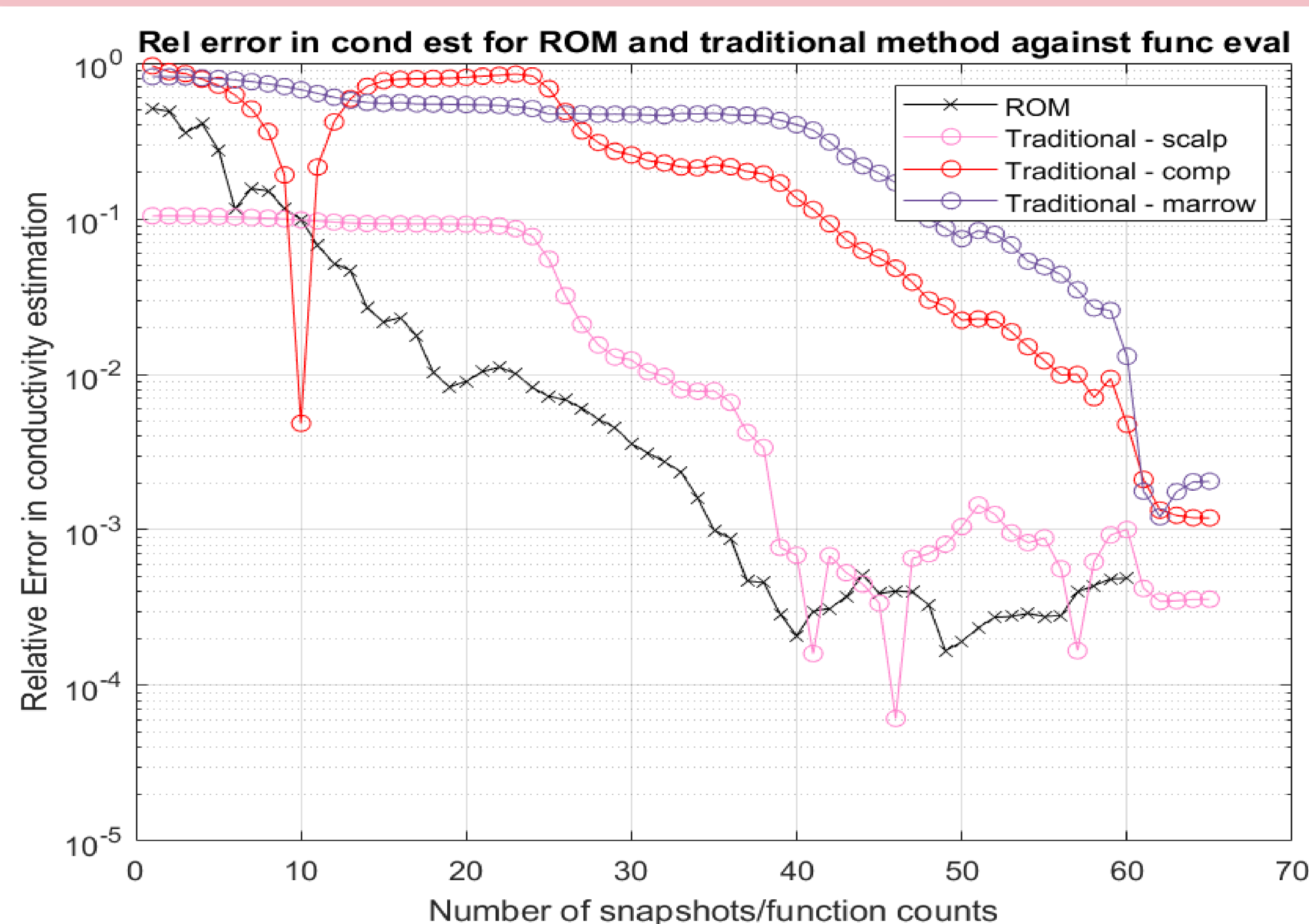
- Realistic head model using Colin27 with 133 electrodes placed on the scalp. Z set to 5 Ωm^2 for each electrode.
- ROM model trained until the maximum bound reaches a relative error of 5×10^{-3} for a range of conductivity values.
- Compared with the traditional method estimating for 3 conductivities simultaneously.
- Std of noise added was $0.82 \mu\text{V}$.

Experiment

- All conductivities estimated to desired accuracy with fewer full order solutions needed.
- Unexpected result: spongiform bone is estimated to a lower error.

Results

Full Order Model Conductivity Estimation	≈ 60 full order solutions needed for 10^{-2} overall RE
Reduced Order Model Training and Conductivity Estimation	≈ 20 full order solutions needed for 10^{-2} overall RE



What else can we do/what's next?

- Estimate conductivity with all pairs simultaneously.
- Include the anisotropy of the conductivity.
- Efficiently recreate any electrode injection pattern.
- Simultaneous conductivity estimation and source localization for EEG.

EIT Process

Generate a head model with individual geometry and conductivity parameter $\sigma = [\sigma_{\text{scalp}}, \sigma_{\text{skull}}, \sigma_{\text{CSF}}, \sigma_{\text{brain}}]$. Inject and extract current from two electrodes and measure potential from multiple others. A specific tissue is then estimated using the following algorithm (The Inverse Problem):

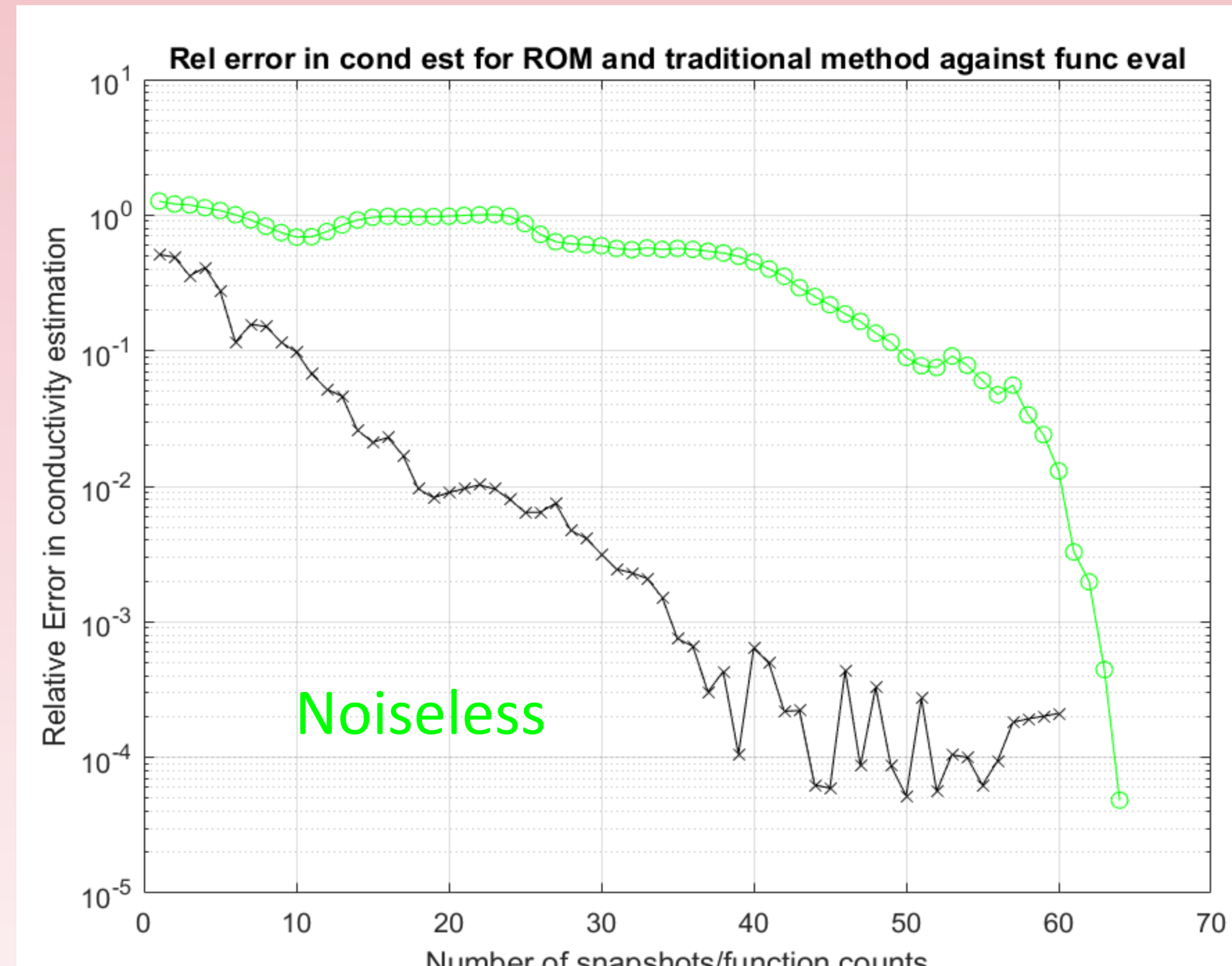
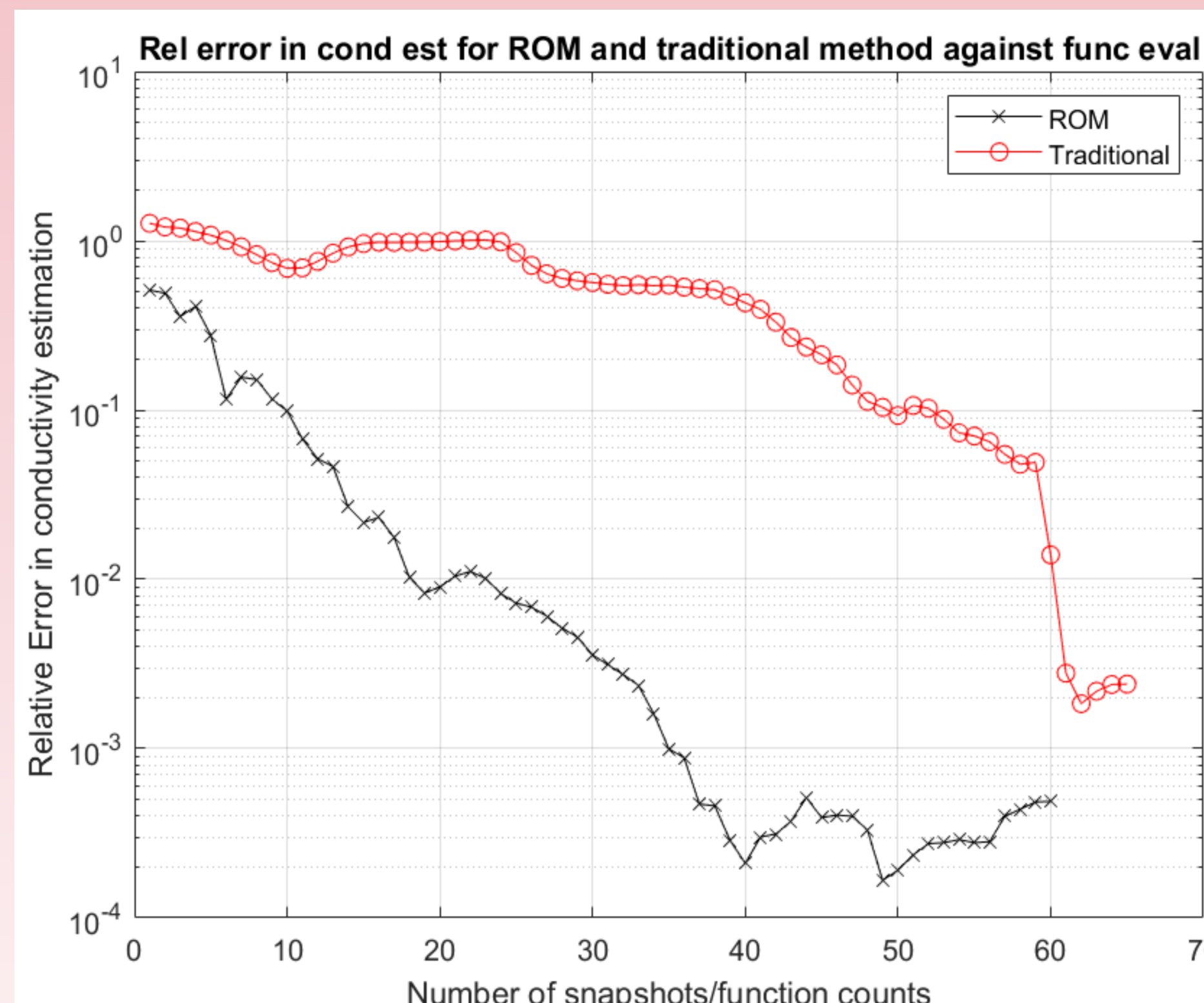
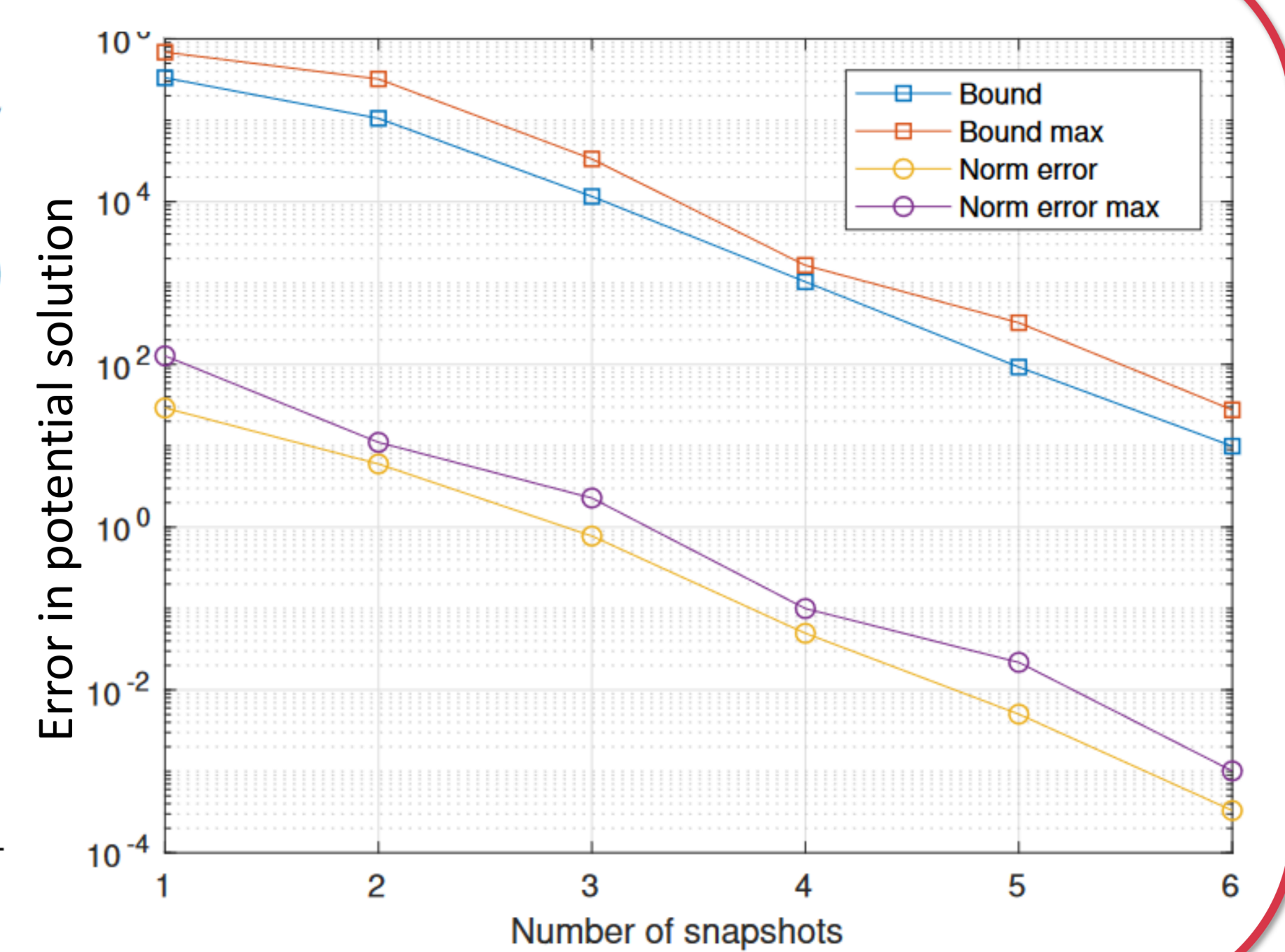
- Calculate the forward problem for specific σ
- Compare with measured values
- Change conductivity value of a tissue and update
- Repeat all steps until convergence on a conductivity
- Repeat for all electrode pairs and average results

This takes many hours/days for traditional method with a detailed head model!

Reduced Order Modelling

ROM allows us to generate a reduced basis space from a number of 'snapshots' of the full order solution which in turn allows us to create a solution manifold. We can then extract an approximate version of ANY full order solution called the reduced basis solution [3]. The snapshots are chosen based on a greedy algorithm that uses a computationally cheap error bound. This method also requires an affine decomposition of the stiffness matrix.

$$\begin{aligned} \text{Conductivity parameter} \quad A_h(\mu) &= \sum_{q=1}^{Q_a} \theta_a^q(\mu) A_h^q \\ \text{Reduced basis solution} \quad A_N(\mu) \mathbf{u}_N(\mu) &= b_N(\mu) \\ \text{Transformation matrix of reduced basis functions} \quad A_N(\mu) &= \mathbb{V}^T A_h(\mu) \mathbb{V} \\ \text{Residual of the RB solution} \quad b_N(\mu) &= \mathbb{V}^T b_h(\mu) \\ \text{Error bound} \quad \Delta_N(\mu) &= \frac{\|r(\cdot; \mu)\|_{V_h'}}{\beta_h(\mu)} \\ \text{Inf-sup constant} \end{aligned}$$



References

- [1] H. McCann et al. Variation in reported human head tissue electrical conductivity values, 2019.
- [2] M. Fernandez-Corazza et al. Skull modeling effects in conductivity estimates using parametric electrical impedance tomography, 2018.
- [3] A. Quarteroni et al. Reduced Basis Methods for Partial Differential Equations. Springer, 2016.